RAN-2503000504033001

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S. Y. B. Sc. (Sem.-IV) Examination September - 2025

MH-MJ-3-403-Number Theory-I (Mathematics-X)

Time: 2 Hours] **[Total Marks: 50**

સૂચના : / Instructions

(૧) નીચે દર્શાવેલ ☛ નિશાનીવાળી વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the details of ☛ signs on your answer book

Name of the Examination:

☛ **S. Y. B. Sc. (Sem.-IV)**

Name of the Subject :

☛ **MH-MJ-3-403-Number Theory-I ((Mathematics-X)**

Subject Code No.: **2503000504033001**

Seat No.:

Student's Signature

(2) All questions are compulsory.
(3) Figures to the right indicate marks of the questions.
(4) Follow usual notations.

Q.1. Answer the following questions (Any Five). **(10)**

(1) If gcd(a,b)=l and gcd(a, c)=1 then prove that gcd(a, bc)=1

(2) For any arbitrary integer a, verify that $2|a(a+1)$

(3) For any integer a, prove that $a^4 \equiv 0 \text{ or } 1 \pmod 5$

(4) If $a^3 \equiv 6 \pmod 7$ then find the integer a.

(5) Define: Diophantine equation.

(6) The integer 701 is prime or composite? Justify.

(7) Find lcm(306,657)

(8) Prove that the only prime of the from $n^3 - 1$ is 7.

Q.2. Answer the following (Any two). **(10)**

(1) For positive integers a and b, prove that $\gcd(a,b) \cdot lcm(a,b) = a \cdot b$

(2) Prove that cube of any integer has one of the form 9k, 9k+l or 9k+8.

(3) Find the integers x and y such that $\gcd(56,72) = 56x + 72y$ by using Euclidean Algorithm.

Q.3. **Answer the following (Any two).** **(10)**

(1) Show that the linear Diophantine equation $ax + by = c$ has a solution if and only if d / c *where* $d = \gcd(a, b)$

(2) Determine all the solution in the positive integer of the Diophantine equation $18x + 5y = 48$.

(3) If $p \geq q \geq 5$ and p, q are both prime then prove that $24 / p^2 - q^2$.

Q.4. **Answer the following (Any two).** **(10)**

(1) Prove that any composite integers ‘a’ will always have a prime divisor p such that $p \leq \sqrt{a}$.

(2) If p is a prime and $p|a_1 \cdot a_2 \cdot a_3 \cdots a_n$ then prove that p/a_k for some k, $1 \leq k \leq n$.

(3) If $p \geq 5$ is prime then prove that $p^2 + 2$ is composite.

Q.5. **Answer the following (Any two).** **(10)**

(1) Let $p(x) = \sum_{k=0}^{m} c_k x^k$ be a polynomial function with integer coefficients. If $a \equiv b \pmod{n}$ then prove that $p(a) \equiv p(b) \pmod{n}$

(2) Show that $89|2^{44} - 1$ and $97|2^{48} - 1$

(3) Without performing the divisor, determine whether the integer 58763120 is divisible by 9 and 11.